RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. SIXTH SEMESTER EXAMINATION, MAY 2019

THIRD YEAR [BATCH 2016-19]

Date : 27/04/2019 **MATHEMATICS (Honours)**

Time : 11 am – 2 pm **Paper :** VIII Full Marks : 70

[Use a separate Answer Book for each Group]

Group - A

(*Answer **any five** questions*) [5×6]

1. A heavy uniform string, of length l, is suspended from a fixed point A, and its other end B is pulled horizontally by a force equal to the weight of a length 'a' of the string. Show that the horizontal and vertical distances between A and B are $a\sinh^{-1}\frac{l}{a}$ and $\left(\sqrt{l^2+a^2}-a\right)$.

2. A ring of weight wb, is attached to the middle point C of a string of length l and weight lw, which hangs symmetrically over two smooth pegs in the same horizontal line, the ends of the string hanging vertically. Show that the parameter c of the catenary is given by the equation $b+l=e^{\frac{a}{c}}\left[b+\sqrt{4c^2+b^2}\right]$, where 2a is the distance between the pegs.

3. A heavy elliptic disc, placed on a rough table and acted on by a horizontal force P, begins to turn about a focus. If its weight be uniformly distributed over the area, prove that the force must act along an ordinate at a distance $\frac{2a}{3}\cdot\frac{\left(1-e^2\right)}{e}$.

4. Forces X,Y,Z act along the three lines given by the equation y = 0, z = c; z = 0 , x = a; x = 0, y = b; prove that the pitch of the equivalent wrench is $\frac{aYZ+bZX+cXY}{\left(X^2+Y^2+Z^2\right)}$. If the wrench reduces to a single force, show that the line of action of the force must lie on the hyperboloid $(x-a)(y-b)(z-c)-xyz=0$. [3+3]

5. A system of coplanar forces acts on a rigid body and the moments of the system about three non-collinear points in the plane are α,β,γ. Prove that

   i) if α,β,γ are not all equal, the system is equivalent to a single force;

   ii) If $\alpha=\beta=\gamma\neq 0$, the system is equivalent to a couple;

   iii) If $\alpha=\beta=\gamma=0$, the system is in equilibrium. [6]

6. Four uniform rods AB, BC, CD and DE, each of length 'a' and weight W are smoothly jointed together at their ends B,C and D and the ends A, E are smoothly jointed to fixed points at distance 2a apart in the same horizontal line. If AB, BC makes angles θ and ϕ respectively with the horizontal when the system hangs in equilibrium, show by the principle of Virtual Work

that $3\cot\theta = \cot\phi$. B and D are connected by an inextensible string of length a. Find the tension of the string. [6]

7. A square lamina rests with its plane perpendicular to a smooth wall one corner being attached to a point in the wall by a fine string of length equal to the side of the square. Find the position of equilibrium and show that it is stable.

8. A stiff wire in the form of a parabola rests on the horizontal ground with its plane vertical. The centre of gravity of the wire is on the axis of the parabola at a distance h from the vertex, and the latus rectum is 4a. Prove that, if h>2a, there is a position of equilibrium in which the axis makes an angle $\tan^{-1}\left(\frac{a}{h-2a}\right)^{1/2}$ with the horizon and that this position of equilibrium is stable. [6]

Group - B

(*Answer **any two** questions*) [2×10]

9. a) Draw a switching circuit diagram for the Boolean function $f(x_1, x_2, x_3) = x_1.x_2' + x_2 + x_3$ and find its corresponding conjunctive normal form. [4]

b) What do you understand by machine Language? How does it differ from high level languages? What is the function of a compiler? How does it differ from an interpreter? [4]

c) What are the basic differences of a for and a while loop? [2]

10. a) A function f is defined as follows:

$$\begin{aligned} f(x) &= -3, x < -3 \\ &= x, -3 \le x \le 3 \\ &= 3, x > 3 \end{aligned}$$

Write a program in C to define the above function and also to evaluate $z = f(a) + \frac{f(b)}{f(a+b)}$ for $a = -4.0$ to 4.0 with increment 1.0 and b = 1.0 [5]

b) Let $(B, +, \bullet)$ be a Boolean ring. Define binary operations $\oplus$ and $\odot$ on B as follows

$$a \oplus b = a + b + a \cdot b$$
$$a \odot b = a \cdot b$$

Also, define an unary operation $'$ on B as $a' = I + a$ where $I \in B$ is the unit element of the ring B. Show that $(B, \oplus, \odot, ')$ is a Boolean algebra. [5]

11. a) Let $(B, +, \cdot, ')$ be a finite Boolean algebra. Show that any non-zero element $x \in B$, can be expressed uniquely as the sum of atoms. Further, show that x dominates all the atoms in that unique expression. [4]

b) The shortest distance of a point (a,b) from a straight line $Ax+By+C=0$ is given by $d=\frac{|Aa+Bb+C|}{\sqrt{A^2+B^2}}$. Write a C-program to accept a point and the coefficients of a straight line and find the shortest distance of the point from this line. [3]

c) Write an algorithm to find the H.C.F. of two distinct positive integers by Euclid's algorithm. Also indicate the case when the numbers are co-prime. [3]

<u>Group – C</u> [Graph Theroy]

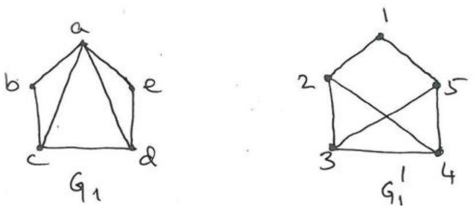
(*Answer **<u>any four</u>** questions*) [4×5]

12. a) Show that the maximum number of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$.

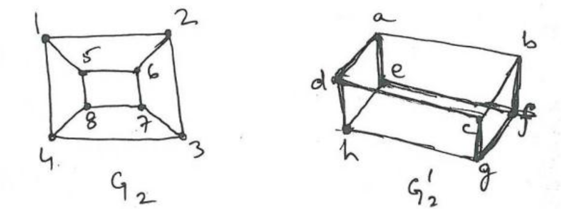
b) Define isomorphism of graphs with an example.

c) Find whether the following graphs are isomorphic? [2+1+2]

i)

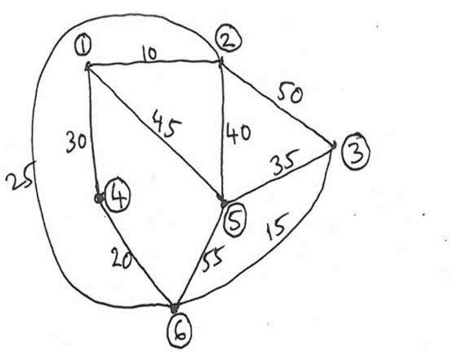


ii)



13. a) Define spanning tree and minimal spanning tree of a weighted connected graph.

b) Using Prim's algorithm find the minimal spanning tree for the following graph: [2+3]



14. Define a binary tree. If n and p be the number of vertices and number of pendant vertices respectively in a binary tree, then show that n is odd and $p=\frac{n+1}{2}$. [1+2+2]

15. Prove that a connected planar graph with n vertices and e edges has $e-n+2$ regions. [5]

16. a) Write the common properties of K_5 and $K_{3,3}$. [3]

b) Using Euler's formula, show that Kuratowski's second graph is nonplanar. [2]

17. Prove that a connected graph G is an Euler graph if and only if it can be decomposed into circuits. [5]

Group – C [Topology]

(*Answer* ***any four*** *questions*) [4×5]

18. a) Find an open cover of $(0,1)\cap(\mathbb{R}-\mathbb{Q})$ having no finite subcover.

b) Give an example of a continuous map $f:\mathbb{Q}\to\mathbb{R}$ which can not be extended to a continuous map from $\mathbb{R}$ to $\mathbb{R}$. Justify your answer. [2+3]

19. Let (X,τ) and (Y,τ_1) be two topological spaces and $f:(X,\tau)\to(Y,\tau_1)$ be a mapping. If f is continuous at $a\in X$, then show that for every sequence $\{x_n\}_{n\in\mathbb{N}}$ in X converging to $a\in X$, the sequence $\{f(x_n)\}_{n\in\mathbb{N}}$ converges to f(a) in Y. Does the converse hold? Support your answer. [2+3]

20. Show that every topological space is a subspace of a separable space.

21. Let $A=\{(x,y)\in\mathbb{R}^2: x^2+y^2=1\}$ and $B=\left\{(x,y)\in\mathbb{R}^2:\frac{x^2}{4}+\frac{y^2}{9}=1\right\}$

Are A,B homeomorphic as subspaces of $\mathbb{R}^2$? Justify your answer.

22. Suppose $f:(\mathbb{R},\tau_c)\to(\mathbb{R},\tau)$ is defined by $f(x)=1+x, \forall x\in\mathbb{R}$, where τ_c and τ denote respectively the cocountable topology and the usual topology on $\mathbb{R}$. Verify whether f is open, closed and continuous.

23. a) Let $X=\{a,b,c,d\}$ and $\tau=\{\phi,X,\{b\},\{c\},\{a,b,c\},\{b,c\}\}$. Find η_b in (X,τ), where η_b is the neighbourhood system at b. [2]

b) Let τ be the co-finite topology on $\mathbb{R}$. Prove that every infinite subset of $\mathbb{R}$ is dense in $(\mathbb{R},\tau)$. [3]

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